Policy, Research, and External Affairs

WORKING PAPERS

Welfare and Human Resources

Population and Human Resources
Department
The World Bank
April 1991
WPS 647

Growth Rates and Aggregate Welfare

An International Comparison

Nanak Kakwani

An alternative procedure for calculating aggregate growth rates is developed, one more suitable for comparing different countries' welfare.

Policy, Research, and External Affairs

WORKING PAPERS

Welfare and Human Resources

WPS 647

This paper — a product of the Welfare and Human Resources Division, Population and Human Resources Department — is part of a larger effort in PRE to develop measures suitable for tracing a country's development over time, with special emphasis on the welfare of the population. Copies are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Brenda Rosa, room S9-137, extension 33751 (62 pages).

Kakwani explores the relationship between growth rates and changes in welfare, using alternative procedures for measuring growth.

The Bank and other organizations commonly compute growth rates by fitting a least-squares linear trend line to the logarithmic values of economic indicators for a period. B . is the least-squares procedure appropriate for measuring people's economic welfare over time?

Kakwani develops a conceptual framework for deriving an aggregate growth rate from a welfare function defined in terms of levels of per capita incomes in different years. Using this function, he derives the welfare implications of alternative procedures for estimating growth. The new procedure captures all the desirable properties of a welfare function.

Kakwani also deals with the issue of aggregating growth rates over countries. If one is interested in judging the growth rates for all countries in Africa, for example, there are two drawbacks to using the country classifications developed for the *World Development Report*. First, the method depends on exchange rates with changes in welfare. Second, the *World Development Report* gives greater weight to the growth rates of richer countries (not necessarily the most populated ones), which is highly questionable for measuring welfare.

Kakwani proposes an alternative procedure for calculating aggregate growth rates, one more suitable for comparing different countries' welfare.

Growth Rates and Aggregate Welfare:
An International Comparison

by Nanak Kakwani*

TABLE OF CONTENTS

LIST OF TABLES

* Welfare and Human Resources Division, the World Bank and Department of Econometrics, The University of New South Wales, Australia. The author gratefully acknowledges Martin Ravallion, Jacques van der Gaag, .. Subbarao and Ravi Kanbur for their helpful comments on this paper. I am most grateful to Bela Balassa for carefully reading the paper and making extremely useful comments.

Table of Contents Continued:

LIST OF GRAPHS

Growth Rates and Aggregate Welfare:
An International Comparison

1. INTRODUCTION

The gross national product (GNP) per head and related income measures are widely used to appraise the economic well-being of people living in different countries. These measures have been subject to much criticism for their failure to give any indication of how the total output of a country is distributed among its people.[1] Recently, many economists, the most notable of them being Sen (1985), have been concerned with whether these aggregated income measures reflect the well-being of people. Despite these criticisms, the aggregate income statistics continue to be widely used to distinguish rich and poor countries, perhaps because of readily available national accounts for most countries of the world.

The present paper is concerned with the measurement of growth rates of various broad economic indicators. It is clearly important to determine whether and to what extent people are becoming better or worse off over time. Not surprisingly therefore, magnitudes of growth rates are always the focus of attention when economists discuss alternative strategies of economic development. However, how these growth rates should be computed is seldom discussed in the economic literature.[2]

---

[1]See Sen (1973, 1974) and Kakwani (1981) who have attempted to derive alternative welfare measures which take into account both the size and distribution of income.

[2]Although, we recognize that the reliability and comparability of national accounts data are subject to serious limitations, this issue has not been considered in this paper. However, the World Bank puts enormous efforts in improving these data on a continual basis.

There may be several purposes for measuring growth rates. One purpose may be to see how the structure of an economy has been changing over time; whether the economy has moved from a lower (higher) to a higher (lower) growth path. Although we deal with this issue in the paper, our main focus is on the welfare aspect of growth rates. The main contention of this paper is that if there is an improvement (deterioration) in the growth rate during a period (for instance, 1970 to 1979), then it should imply a higher (lower) welfare level for that period.

Thus, there must exist a one-to-one relationship between growth rates and changes in welfare levels. This paper explores this relationship using alternative growth procedures.

Growth rates of several economic indicators routinely appear in World Bank documents and are given wide circulation in publications such as the World Development Report and the World Bank Atlas. These growth rates are computed by fitting a least-square linear trend line to the logarithmic values of the indicators in the relevant period. This method is used widely not only by the World Bank but also by other international organizations and individual countries.

The issue to be discussed in this paper is whether the widely used least-squares procedure is the appropriate one if our objective is to see how the economic welfare of people has changed over a period of time. The paper develops a conceptual framework to derive the aggregate growth rate from a welfare function defined in terms of levels of per capita incomes in different years. Using this framework, welfare implications of alternative growth estimation procedures are derived. A new procedure is suggested which captures all the desirable properties of a welfare function.

The present paper also deals with the issue of aggregating growth rates over countries. The issue is important because we often wish to compare the growth performance of countries grouped according to some economic or demographic criteria. For instance, if we are interested in judging the growth performance in Africa, then we need to aggregate the growth rates of all countries in Africa. The World Development Report (WDR) presents such information for a wide range of country classifications. The WDR method has two drawbacks. First, the method depends on exchange rates and it is questionable to equate changes in exchange rates with changes in welfare. Second, it gives greater weight to the growth rates of richer countries (not necessarily the most populated ones), which from the welfare point of view is highly objectionable. The paper proposes an alternative procedure for calculating aggregate growth rates more suitable for welfare comparisons.[3]

The methodology developed in the paper is applied to analyze growth rates of per capita GNP of eighty-three developing countries during the 1970-87 period.

2. THE LEAST SQUARE GROWTH RATE

Let $x_1, x_2, \ldots, x_n$ be the values of an economic indicator (such as GDP or GNP per head) given for n periods. These values are in constant prices so that the effect of inflation has been taken into account.[4] The question we

[3]The World Bank extensively uses per capita GNP and its growth rate for its operational purposes which relate to decisions about member countries' eligibility for beneficial borrowing terms and other advantages. Clearly then, the welfare should be the main criterion for assessing the suitability of the procedures used.

[4]This paper does not address the issue of measuring the inflation rate which poses complex technical problems, not yet fully and unequivocally resolved.

ask is what is the most appropriate single growth rate of x for these n years. The most commonly used procedure is the least-squares method. The least-squares growth rate R is estimated by fitting a trend line

$$\log x_t = \alpha + \beta t + e_t \tag{2.1}$$

where t varies from 1 to n; α and β are the parameters to be estimated and e_t is the error term. This Equation is equivalent to the logarithmic transformation of the compound growth rate equation

$$x_t = x_0 (1 + R)^t \tag{2.2}$$

where $\alpha = \log x_0$, x_0 being the value of x in the base period and $\beta = \log(1 + R)$.

If $\hat{\beta}$ is the least-squares estimate of β, the estimated growth rate $\hat{R}$ is obtained as antilog $(\hat{\beta}) - 1$.

It can be shown that

$$\hat{\beta} = \log(1 + \hat{R}) = \frac{\sum_{t=1}^{n} \log x_t \, (t - \bar{t})}{\sum_{t=1}^{n} (t - \bar{t})^2} \tag{2.3}$$

where $\bar{t} = \frac{(n + 1)}{2}$. Let

$$r_t = \frac{x_t - x_{t-1}}{x_{t-1}} \tag{2.4}$$

be the growth rate for the t^{th} year, where t varies from 2 to n. The question then arises as to how the total growth rate $\hat{R}$ (for the entire period) is related to the growth rates in each year. To answer this question, substitute (2.4) into (2.3), which after complicated algebraic manipulations leads to

$$\beta = \log(1 + \hat{R}) = \sum_{t=2}^{n} w_t \log(1 + r)_t \tag{2.5}$$

where

$$w_t = \frac{6\ (t - 1)\ (n + 1 - t)}{n\ (n + 1)\ (n - 1)} \tag{2.6}$$

such that $\sum_{t=2}^{n} w_t = 1.0$. Equation (2.5) implies that the growth rate for the whole period is approximately equal to the weighted average of the growth rates in each year. It can be seen that the weight w_t increases with t until $t = \frac{n}{2}$ and then it decreases.

Thus, the least-squares growth rate (which will be referred to as the LSGR procedure) gives maximum weight to the growth rates in the middle of the time period (over which it is computed). The lower weights are given to the growth rates in the beginning and at the end of the time period.

We have demonstrated that the LSGR procedure which appears to be a mechanical one of fitting a time trend has an intuitive interpretation. The question then naturally arises whether the weighting scheme implied by it is generally acceptable. Are there other alternative procedures which are intuitively more appealing? We discuss alternative procedures in the next section.

3. ALTERNATIVE PROCEDURES

In the previous section we demonstrated that the LSGR procedure gives different weights to growth rates in different years. One obvious alternative is to give equal weights to all growth rates in (2.5), i.e., $w_t = \frac{1}{(n-1)}$.

Denoting the estimated growth rate by this procedure as $\hat{R}_1$ Equation (2.5) gives

$$\log(1 + \hat{R}_1) = \frac{1}{(n-1)} \sum_{t=2}^{n} \log(1 + r_t) \tag{3.1}$$

which can further be simplified to

$$\hat{R}_1 = \left[\frac{X_n}{X_1}\right]^{\frac{1}{n-1}} - 1 \tag{3.2}$$

which uses only the first and the last observations of the period. Clearly, this procedure (which will be referred to as geometric mean growth rate GMGR) is inappropriate because the total growth rate obtained by it is completely insensitive to the fluctuations in the values of x between the end points of the whole period. If we are interested in long term growth rates, this procedure will yield particularly misleading results.

The time trend equation in (2.1) was obtained from the compound growth rate equation (2.2), where $\alpha = \log x_0$ and $\beta = \log(1 + R)$, R being the growth rate. The parameters α and β were estimated by the least-squares method. Since the initial value x_0 is known, an alternative procedure for estimating R will be to impose the restriction that $\alpha = \log x_0$.

Since we are measuring the total growth rate from $x_1, x_2, \ldots, x_n$, our initial value is x_1 and, therefore, we impose the condition that the time trend equation in (2.1) passes through x_1. Therefore, Equation (2.1) becomes

$$(\log x_t - \log x_1) = \beta\,(t - 1) + e_t \tag{3.3}$$

where $t = 2, 3, \ldots, n$. Applying the least-squares method to (3.3) yields

$$\beta = \log(1 + \hat{R}_2) = \frac{\sum_{t=2}^{n} (\log x_t - \log x_1)\,(t - 1)}{\sum_{t=2}^{n} (t - 1)^2} \tag{3.4}$$

where $\hat{R}_2$ is the estimated total growth rate by this procedure. After complicated algebraic manipulations, (3.4) yields

$$\log(1 + \hat{R}_2) = \sum_{t=2}^{n} w_t \log(1 + r_t)$$

where

$$r_t = \frac{(x_t - x_{t-1})}{x_{t-1}}$$

and

$$w_t = \frac{3\,[n(n-1) - (t-1)\,(t-2)]}{n\,(n-1)\,(2n-1)} \tag{3.5}$$

such that $\sum_{t=2}^{n} w_t = 1.0$. This procedure will be referred to as the restricted least-squares growth rate (RLSGR).

It can be seen that w_t in (3.5) is a decreasing function of t, which means that RLSGR gives maximum weight to the growth rates in the beginning of the period. The least weight is given to the most recent growth rates. This weighting scheme may be considered unacceptable if we are most interested in the recent growth rates. Then, an alternative weighting scheme will be the one in which w_t is a monotonically increasing function of t. A simple weight function satisfying this property is

$$w_t = \frac{2t}{(n^2 + n - 2)} \tag{3.6}$$

so that $\sum_{t=2}^{n} w_t = 1.0$. Denoting the estimated total growth rate by $\hat{R}_3$, Equation (3.6) gives

$$\log(1 + \hat{R}_3) = \frac{2\sum_{t=2}^{n} t \log(1 + r_t)}{(n^2 + n - 2)} \tag{3.7}$$

This procedure will be referred to as increasing weight growth rate (IWGR).

Finally, we present the most simple way of computing the total growth rate:

$$\hat{R}_4 = \frac{1}{(n - 1)} \sum_{t=2}^{n} r_t \tag{3.8}$$

which is nothing but the simple average of yearly growth rates. This will be called as arithmetic mean growth rate (AMGR).

Note that this procedure gives exactly the same weight to growth rates in every year. Thus, it assumes complete symmetry between negative and positive growth rates which means that the adverse impact of a negative growth rate in a year cancels out by a positive growth rate of the same magnitude in another year. It must be further noted that GMGR gives the same weight to $\log(1 + r_t)$ for all t which is not the same thing as giving equal weights to each r_t. Thus, the two methods differ with respect to their welfare implications. This issue will be discussed further in Section 6.

4. GROWTH RATES OF GNP PER CAPITA IN SEVENTY-SEVEN DEVELOPING COUNTRIES, 1970-87

The alternative procedures for computing growth rates (discussed in the previous section) will now be applied to the data from eighty-three developing countries. These data were taken from the World Bank Data Files by means of the ANDREX system. Gross national product (GNP) per capita was used to compare the economic performance of countries over the period 1970-87. All growth rates were computed in constant prices using the local currencies. The numerical results are presented in Table 1. The first column in the Table provides growth rates computed by the least-squares procedure which gives maximum weight to growth rates around the 1979 year. The remaining four columns in the Table present growth rates computed by four alternative procedures proposed in the paper.

It can be seen from the Table that the growth performance of countries vary widely. Most African countries have performed extremely badly. In a large number of these countries, growth in GNP was less than the growth in population. The deterioration in growth rates is even worse in the low-income

African countries. Such a widespread decline in per capita GNP must certainly have serious implications for the living conditions of the majority of people in Africa, many of whom were already living below the subsistence standard. The countries in Africa which have performed well are Botswana, Cameroon, Congo, Lesotho, and Mauritius.

It is evident that the Asian group of countries have performed relatively better than the rest of the world during the period 1970-87. China, the most populated country in the world has demonstrated the extremely impressive growth performance. The countries which can be classified as poor performers are Nepal, Bangladesh, and the Philippines.

Prior to 1970, a large number of the countries in the Central and South American region enjoyed high growth rates. This performance deteriorated considerably in the 1970-87 period. During 1980-87, almost all countries experienced negative growth rates in per capita GNP. These countries were most severely affected by the debt crisis induced by high interest rates and declining demand for their exports.

The numerical results in the Table also show that growth rates computed by alternative procedures vary substantially for a large number of countries. These differences occur because of the differences in weighting schemes implied by each method. Most of the Latin American countries performed well in the early seventies and extremely badly in the eighties. Any procedure which gives higher weight to the growth rates in the beginning of the period would show higher values of the total growth rate in these countries. This is quite evident from the numerical results in the Table. In 18 out of 22 Central and South American countries, RLSGR (for which the weight given to growth rates decreases monotonically with time) shows higher values of the total growth rate than the other alternative procedures. According to RLSGR, Brazil's per capita GNP grew at an annual rate of 4.33 percent during the 1970-87 period but LSGR and IWGR3 procedures showed the growth performance of only 2.84 and 2.58 percent, respectively during the same period.

India's growth performance was considerably better in the 1980s than that in the 1970s. Accordingly, IWGR3 which gives higher weights to growth rates in the most recent period should show a higher growth performance. This indeed is the case. Bangladesh's growth rate varies from -0.4 percent to 1.07, depending on which procedure is used.

Table 2 summarizes the characteristics of the countries which had negative per capita GNP growth rates during the period 1970-87. It can be seen that the number of countries which experienced negative growth rates, vary from 24 to 35, depending on which procedure is used to compute growth rates. RLSGR yields the smallest number of such countries while IWGR3 the largest number. The difference between the two procedures is that one gives higher weight to the growth rates at the beginning and another at the end of the period. This observation suggests that the overall growth performance of developing countries deteriorated during the 1980s.

The above analysis clearly demonstrates that inferences concerning the economic performance of countries can vary substantially with respect to the procedure employed. It is, therefore, of considerably importance to know which computational procedure is the most appropriate one. The next two sections provide an evaluation of the alternative procedures from the welfare point of view.

Table 1: GROWTH RATES OF GNP PER CAPITA 1970-87
Based on Alternative Procedures

Country	LSGR	GMGR	RLSGR	IWGR	AMGR
A F R I C A*					
Burundi	0.94	0.47	0.55	0.53	0. 9
Benin	0.44	-0.13	0.15	-0.28	-0.05
Botswana	8.49	9.27	9.50	8.73	9.65
Central African Rep.	-0.91	-0.67	0.47	-0.99	-0.60
Cote d'Ivoire	-0.40	-0.44	0.44	-1.50	-0.36
Cameroon	5.21	4.08	3.95	4.62	4.20
Congo	3.15	2.13	3.32	1.02	2.44
Ethiopia	-0.38	-0.14	-0.10	-0.27	-0.09
Gabon	-1.32	-0.53	1.57	-3.31	0.77
Ghana	-2.59	-1.99	-2.28	-1.83	-1.83
Guinea	0.42	0.75	0.82	0.55	0.79
The Gambia	0.73	1.54	2.05	0.64	1.78
Guinea-Bissau	-1.52	-1.51	-1.68	-1.30	-1.22
Burkina Faso	2.28	1.97	1.90	2.15	2.05
Kenya	1.06	1.99	2.90	0.57	2.14
Liberia	-3.27	-3.01	-2.30	-3.96	-2.93
Lesotho	4.02	3.74	5.68	1.48	4.01
Madagascar	-2.91	-2.63	-2.38	-3.03	-2.55
Mali	1.57	1.47	1.55	1.41	1.70
Mauritania	-0.85	-0.92	-1.03	-0.75	-0.77
Mauritius	3.11	4.14	3.90	4.09	4.28
Malawi	0.28	1.51	11.81	0.73	1.69
Niger	-1.83	-2.62	-2.36	-2.67	-2.30
Nigeria	-1.33	-0.99	0.14	-2.48	-0.80
Rwanda	1.69	0.63	1.00	0.54	0.73
Sudan	-0.80	-1.06	-0.10	-2.14	-0.70
Senegal	-0.64	-0.44	-0.61	-0.29	-0.25
Sierra Leone	-0.45	-0.74	-0.29	-1.20	-0.69
Somalia	0.12	0.13	0.39	-0.19	0.70
Togo	-0.84	-0.72	-0.15	-1.46	-0.56
Tanzania	-1.14	-0.89	-0.46	-1.50	-0.87
Uganda	-3.63	-3.32	-3.31	-3.44	-3.13
South Africa	0.06	0.15	0.43	-0.22	0.19
Zaire	-3.10	-2.31	-2.37	-2.49	-2.22
Zambia	-2.79	-3.13	2.33	-4.00	-2.97
Zimbabwe	0.21	0.68	1.46	-0.43	0.88

MIDDLE-EAST AND EASTERN EUROPE

Algeria	8.50	6.23	7.52	5.40	8.27
Egypt	3.79	3.11	3.32	3.08	3.17
Greece	1.97	2.23	2.97	1.25	2.29
Hungary	4.54	4.62	5.68	3.31	4.82
Jordan	4.35	2.86	3.84	2.16	3.10
Morocco	1.62	1.53	1.90	1.11	1.58
Oman	4.48	3.80	3.09	4.92	5.08
Portugal	2.05	2.80	2.92	2.40	2.91
Syrian Arab Republic	2.62	1.92	4.23	-0.64	2.31
Tunisia	2.90	3.22	4.07	2.08	3.32
Turkey	2.08	2.76	2.77	2.52	2.81
Arab Republic of Yemen	6.09	6.18	7.49	4.55	6.39
Yugoslavia	2.90	2.84	3.77	1.72	2.92

ASIA

Bangladesh	1.07	0.10	-0.40	1.04	0.28
Myanmar	2.69	2.07	2.22	2.10	2.10
People's Rep. of China	5.50	5.52	4.49	6.80	5.63
Indonesia	3.99	3.74	4.33	3.08	3.76
India	2.04	1.85	1.49	2.37	1.92
People's Rep. of Korea	6.20	6.62	6.44	6.71	6.71
Sri Lanka	3.24	2.82	2.83	2.95	2.85
Malaysia	3.96	3.66	4.45	2.80	3.71
Nepal	0.71	0.62	0.30	1.04	0.67
Pakistan	2.60	2.22	1.69	3.00	2.24
Philippines	1.24	1.16	2.13	0.01	1.24
Thailand	3.99	3.78	3.73	3.92	3.81

LATIN AMERICA

Argentina	-1.14	-0.58	-0.42	-0.96	-0.47
Bolivia	-1.59	-1.14	-0.07	-2.60	-1.06
Brazil	2.84	3.67	4.33	2.58	3.78
Chile	-0.13	0.18	-0.32	0.71	0.50
Colombia	2.19	2.42	2.80	1.88	2.45
Costa Rica	0.44	1.01	1.40	0.34	1.13
Dominican Republic	1.54	1.68	2.48	0.64	1.75
Ecuador	2.18	2.20	3.87	0.17	2.37
Guatemala	0.21	0.31	1.31	-0.96	0.37
Honduras	0.27	0.30	0.82	-0.34	0.36
Haiti	0.52	0.47	1.07	-0.26	0.54
Jamaica	2.84	-1.99	-1.91	-2.37	-1.83
Mexico	1.76	1.53	2.23	0.75	1.61
Nicaragua	-3.82	-2.89	-2.40	-3.81	-2.46
Panama	1.79	1.75	2.06	1.3	1.80
Peru	-0.35	0.40	0.17	0.4	0.55
Papua-New Guinea	-0.61	-0.02	-0.08	-0.14	0.03
Paraguay	3.52	2.87	4.01	1.69	2.99
El Salvador	-1.22	-0.66	-0.06	-1.59	-0.55
Trinidad and Tobago	1.50	0.45	1.99	-1.08	0.86
Uruguay	0.95	1.03	1.15	0.85	1.16
Venezuela	-1.74	-1.34	-1.08	-1.79	-1.27

Table 2: CHARACTERISTICS OF COUNTRIES WITH NEGATIVE PER CAPITA GROWTH RATES 1970-87

Regions	Total number of countries	Countries with negative growth rates				
		LSGR	GMGR	RSLGR	IWGR	AMGR
Africa	36	19	20	16	23	19
Middle East and Eastern Europe	13	0	0	0	1	0
Asia	12	0	0	1	0	0
Latin America	22	9	7	7	11	6
All Developing Countries	83	28	27	24	35	25

5. WELFARE INTERPRETATION OF AGGREGATE GROWTH RATE

Let $r_2, r_3, \ldots, r_n$ be the (n-1) growth rates calculated over the period from 1 to n years, we define the aggregate growth rate R as a unique function of each year's growth rates:

$$R = R\ (r_2,\ r_3,\ \ldots,\ r_n), \tag{5.1}$$

satisfying certain desirable properties. Note that index R is unaltered if each value of x is altered by the same proportion. Thus, R is scale-independent implying that x can be measured in any currency; (in US dollars or in constant local currency).

The question posed is what welfare interpretation can be given to index R. The social welfare function as introduced by Bergson in 1938 and subsequently developed by Samuelson in 1947 is generally defined over a set of individuals. Following the same idea, one can define a welfare function of an individual or a group of individuals over a period of time. For instance, $\underset{\sim}{x}$

= $(x_1, x_2, \ldots, x_n)$ is a vector of values of a country's economic indicator (such as GDP on GNP per head) for n years, we then define a welfare function of that country over n years as:

$$W(\underset{\sim}{x}) = W(x_1, x_2, \ldots, x_n) \tag{5.2}$$

such that $\frac{\partial W}{\partial x_i} > 0$ for all i. Further, if we require that a wide fluctuations in yearly per capita GNP rates is undesirable because it creates uncertainty in the economy, then we should restrict $W(\underset{\sim}{x})$ to be quasi-concave in the yearly per capita incomes.

The question we ask is what is the equivalent level of GDP or GNP per head given to the country in each year (the same amount) which would result in the same level of welfare as the present incomes in n years. Let this equivalent level of GNP per head be x^*, then

$$W(x^*, x^*, \ldots, x^*) = W(x_1, x_2, \ldots, x_n) \tag{5.3}$$

must hold. Note that this idea of equivalent level of per capita GNP is similar to the concept of equally distributed equivalent level of income introduced by Atkinson (1970).

x^* is a unique function of income levels $x_1, x_2, \ldots, x_n$ derived from a welfare function and thus it measures the welfare level of the country for a period of n years. Since the welfare function W is quasi-concave, x^* will be less than the arithmetic mean of $x_1, x_2, \ldots, x_n$. The larger the yearly fluctuations in income, the greater will be the deviation of x^* from the mean.

Suppose we wish to compare the welfare levels in two periods; the first period consists of n years with income vector $\underset{\sim}{x}$ and the second period m years with income vector $\underset{\sim}{y}$. Let y^* be the equivalent level of per capita income in the second period. Then the index

$$\eta = \frac{(y* - x*)}{x*} \tag{5.4}$$

will measure the percentage change in welfare from period 1 to period 2. This index is clearly invariant with respect to any linear positive transformation of the welfare function. The positive (negative) value of η would imply an improvement (deterioration) in the welfare in period 2 over period 1. We would apply this index to see how the welfare level has changed in the 1980s compared to that in the 1970s in 83 developing countries.

Let r_t be the growth rate of a country in the year from (t-1) to t, then by definition, $x_t = x_{t-1}(1+r_t)$ must hold. On substituting sequentially, x_t in terms of x_1 will be given by

$$x_t = x_1\ (1 + r_2)\ (1 + r_3)\ \ldots\ (1 + r_t) \tag{5.5}$$

which on substituting in (5.2) gives

$$W(\underset{\sim}{x}) = W[x_1, x_1(1 + r_2), \ldots x_1(1 + r_2)(1 + r_3) \ldots (1 + r_n)] \qquad (5.6)$$

Similar to the idea of equivalent per capita GNP, we may introduce a new concept to be called equivalent uniform growth rate. It is the uniform growth rate which would result in the same level of welfare as the present incomes in n years. Let R be the equivalent uniform growth rate, then the welfare level obtained by this growth rate;

$$W[x_1, x_1(1 + R) \ldots x_1(1 + R)^2 \ldots x_1 (1 + R)^{n-1}] \qquad (5.7)$$

must be equal to the welfare level obtained from the present incomes in n years.

Thus,

$$W[x_1, x_1(1 + r_2), x_1(1 + r_2)(1 + r_3) \ldots x_1(1 + r_2)(1 + r_3) \ldots (1+r_n)]$$
$$= W(x_1, x_1(1 + R), x_1(1 + R)^2 \ldots . x_1(1 + R)^{n-1}) \qquad (5.8)$$

which will give R to be a function of r_2, r_3, ..., r_n if the welfare function W is homothetic.

Thus, using (5.8), R can be derived directly from a welfare function. It is clearly in variant with respect to any positive linear transformation of the welfare function W. If W is homothetic, R will be scale, independent implying that x can be measured in any currency (in US dollars or in constant local currency).

The above formulation suggests that all the growth procedures will have some implicit welfare function. In the next section we would evaluate alternative procedures by examining the welfare function implied by them.

6. AN EVALUATION OF ALTERNATIVE PROCEDURES

Let us consider a general class of welfare functions which are homothetic in incomes:

$$W(\underset{\sim}{x}) = \sum_{t=1}^{n} v_t \, logx_t \tag{6.1}$$

where $\log x_t$ is the welfare or utility enjoyed by a country in year t and v_t is the weight attached to t^{th} year utility such that:

$$\sum_{t=1}^{n} v_{-} = 1.0 \tag{6.2}$$

If R is the uniform growth rate in the period 1 to n years, the welfare level corresponding to this growth rate will be:

$$W(\underset{\sim}{x}) = logx_1 + \log(1 + R) \sum_{t=1}^{n} (t-1) v_t \tag{6.3}$$

Equating (6.1) to (6.3) yields:

$$\log(1 + R) = \frac{\sum_{t=2}^{n} v_t(\log x_t - \log x_1)}{\sum_{t=2}^{n} (t-1) v_t} \tag{6.4}$$

which expresses R as a function of x_1, x_2, ..., x_n. To express R in terms of growth rates r_2, r_3, ..., r_n, we write

$$\log(x_t) - \log(x_1) = \sum_{j=2}^{t} \log(1 + r_j) \tag{6.5}$$

which is derived from $\log(1 + r_t) = \log x_t - \log x_{t-1}$.

Substituting (6.5) into (6.4) gives

$$\log(1 + R) = \sum_{t=2}^{n} w_t \log(1+r_t) \tag{6.6}$$

$$w_t = \frac{\sum_{j=t}^{n} v_j}{\sum_{t=2}^{n} (t-1) v_t} \tag{6.7}$$

such that $\sum_{t=2}^{n} w_t = 1.0$. Thus, we have proved that the class of welfare functions (6.1) results in aggregate growth rate of the whole period being approximately equal to the weighted average of the growth rates in each year. All the growth procedures discussed in Sections 2 and 3 (with the exception of Arithmetic mean growth rate (AMGR)) belong to the class of growth rates in (6.6) which are generated from the welfare function (6.1).

Using (6.2) and (6.7), we can derive implications for welfare weights v_t given w_t. Thus, we can evaluate alternative growth procedures on the basis of welfare weights implied by them.

Let us first consider the geometric mean growth rate (GMGR) procedure for which $w_t = \frac{1}{(n-1)}$ for all t≥2, Equation (6.7) yields the welfare function:

$$W(\underset{\sim}{x}) = \frac{(n-2)}{(n-1)} \log x_1 + \frac{1}{(n-1)} \log x_n \tag{6.8}$$

which shows the welfare weights are zero for the years from 2 to (n-1). Thus, the welfare function is completely insensitive to the utilities enjoyed by the country between the end points of the whole period. Such a welfare function cannot be acceptable because $\frac{\partial W}{\partial x_i} > 0$ for all i, is the minimum requirements of a welfare function, a condition which is clearly violated.

Next we consider the most commonly used least-square growth rate (LSGR) procedure. Using (2.6), we obtained the welfare weights as

$$v_t = \frac{6\ k\ (2t - n-1)}{n\ (n+1)\ (n-1)} \tag{6.9}$$

for $t \geq 2$, k being the constant of proportionality. This equation implies that $v_t = 0$ for $t = \frac{(n+1)}{2}$, and positive (negative) for t being greater (less) than $\frac{(n+1)}{2}$. This is a peculiar welfare function, it gives negative weights to the utilities enjoyed by the countries in the first half of the total period. To highlight this peculiarity, let us consider a simple example of income streams of two countries in five periods; country 1: 100, 200, 300, 400, and 500; country 2: 100, 100, 300, 400, and 500. Country 1 clearly enjoys a greater level of welfare than country 2, because its income in period 2 is twice (incomes in other periods being the same). Therefore, the aggregate growth of country 1 must be higher than that of country 2. The least-squares growth rates for countries 1 and 2 were computed to be 48 and 58 percent, respectively; implying that the higher the growth rate, the lower the aggregate welfare. This possible inverse relationship between aggregate growth rate and aggregate welfare is clearly unacceptable. LSGR is used widely by the World Bank and many other international organizations. Despite its popularity, the procedure is found to be unsuitable from the welfare point of view.

Does the restricted least squares growth procedure (RLGR) have the desirable welfare properties? To answer this question we utilized (3.4) and (3.5) to obtain the welfare weights:

$$v_t = \frac{k\ (t-1)}{n\ (n-1)\ (2n-1)} \tag{6.10}$$

for t>1. These weights are all positive implying that every year's utility has a positive impact on the total welfare. This welfare function is clearly more acceptable. However, weights given to each year's utility are not uniform, it gives least weight to the utility enjoyed by the country in the beginning of the period. The largest weight is given to the income in the most recent year.

The increasing weight growth rate (IWGR) procedure gives (on using (6.7) and (3.7)) the following welfare weights:

$$v_t = -\frac{2k}{n^2 + n-2} \quad for\ 1 < t < n-1$$

$$\frac{2nk}{n^2 + n-2} \quad for\ t = n.$$

which implies that only the income of the most recent year gets the positive welfare weight and incomes in all other years receive negative weight. Again, such a welfare function is highly inappropriate.

Finally, we consider the arithmetic mean growth rate (AMGR) procedure. It can be seen that this procedure does not belong to the class of growth rate procedures in (6.6). However, it is easy to show that the welfare function

$$W(x) = \frac{x_1}{(n-1)} \left[\frac{x_2}{x_1} + \frac{x_3}{x_2} + \ldots + \frac{x_n}{x_{n-1}} \right]$$

will lead to AMGR procedure. This welfare function is also not intuitively appealing because an increase in income in a particular year may lead to a reduction in the total welfare. This can be seen from the two country examples considered above. Country 1 gave an average growth rate of 52.08 percent whereas for country 2, the average growth rate was found to be 64.58 percent, thus implying an inverse relationship between aggregate growth rate and aggregate welfare.

The above analysis suggests that among all the aggregate growth procedures only the restricted least-squares (RLSGR) implies a meaningful welfare function. This is the only procedure which gives positive weights to a country's utility in each year. The question then arises whether this weighting scheme is intuitively meaningful. If not, can we derive an alternative procedure which is more appropriate from the welfare point view? For this, turn to the next section.

7. A NEW PROCEDURE

An evaluation of alternative procedures discussed in the preceding sections suggested that the welfare point of view, RSLGR provides the best estimator of the aggregate growth rate. The welfare weights derived for this procedure are positive. However, the major drawback of this procedure is that it gives different weight to the utilities in different years. It is not clear why should the consumption of a country in year t be given a different weight from that in year t'. Intuitively, an income of $100 should make the same contribution to the total welfare irrespective of which year it is consumed. Or, in other words, the welfare function should be symmetric in utilities levels in different years.

A simple symmetric function given by

$$W(\underset{\sim}{x}) = \frac{1}{n} \sum_{t=1}^{n} \log x_t \tag{7.1}$$

is increasing in incomes, quasi-concave and symmetric. We have adopted a symmetric welfare function because we are unable to provide a reasonable justification for a non-symmetric welfare function.[5] However, our framework is general enough to use any welfare function.

Applying the formula (6.7) on (7.1) immediately gives an aggregate growth procedure

$$\log(1 + R) = \sum_{t=2}^{n} w_t \log(1 + r_t) \tag{7.2}$$

where

$$w_t = \frac{2(n-t+1)}{n\ (n-1)} \tag{7.3}$$

[5]In many economic policy formulations, we discount the future utility which means that the greater weight is given to the current utility compared to that in the future. The main motivation for discounting the future utility is that the individuals prefer to consume in the current period, but if they forgo current consumption, they must be compensated so that they can have a higher consumption in the future. This issue is different from the one we are dealing with here. We are not concerned with the maximization of an individual's utilities. We are dealing only with the measurement of welfare over time given a stream of consumption or income of a country in each period. How individuals should allocate consumption in different years is not our concern.

such that $\sum_{t=2}^{n} w_t = 1.0$.

It can be seen that w_t in (7.3) is a decreasing function of t, which means that the proposed procedure gives maximum weight to the growth rates in the beginning of the period. The least weight is given to the most recent growth rates. Is this weighting scheme appropriate from the welfare point of view? The answer seems to be yes. Consider for instance two situations: first in which a growth rate of 10 percent in the first year and 20 percent in the second year; and in the second situation, a growth rate of 20 percent in the first year and 10 percent in the subsequent year. According to the average growth rate procedure both situations are equally preferred but the proposed procedure would dictate the second situation be preferred to the first. It is easy to see that the income levels in three years (starting with 100) for the first and the second situations are 100, 110, 132, and 100, 120, 132, respectively. The income in year 2 in the second situation is higher than that in the first situation and thus the second situation is welfare superior to the first situation. Clearly then, the higher growth rates in the beginning of the period are preferable to those in the latter periods. Thus, the weights given in (7.3) are meaningful from the welfare point of view.

Since the chosen welfare function is symmetric in income levels, the aggregate growth rate R should be invariant to any permutation of the income levels, $x_2, x_3, \ldots, x_n$. This can be seen by considering the income levels in the three periods, viz, 100, 200, and 300; the aggregate growth rate for these three periods is computed from (7.2) to be equal to 81.7 percent. Suppose that the income 300 occurs in the second period and 200 in the third period. Because of the symmetry of welfare function, this switching of incomes should not affect the total welfare level and, therefore, the aggregate growth rate in the two situations should be the same. It can be verified that this is indeed the case.

Summarizing the above discussion we can say that of all the aggregate growth procedures, the proposed method is the most desirable one. First of all, we have derived it from a welfare function and, therefore, it provides a positive relationship between the aggregate growth rate and the aggregate welfare. If a higher growth rate is preferred to the lower growth rate, then an increase in growth rate should imply a higher level of welfare. Secondly, it is simple to compute; it is equal to the weighted average of the logarithmic function of yearly growth rates.

8. SUB-PERIOD GROWTH RATES

This section addresses the issue of structural change in the economy. The question we ask is whether the economy has moved from a lower (higher) to a higher (lower) growth path. Therefore, we divide the total period into two sub-periods. Let R_1 be the aggregate growth rate of the first sub-period (1 to n_1) and R_2 that of the second sub-period (n_1+1 to n) so that $n_1 \leq n$. The economy has moved into a higher (lower) growth path if $R_2 > R_1$, ($R_2 < R_1$). And, therefore, the compound growth rate equation (2.2) can be rewritten as

$$\begin{aligned} x_t &= x_1(1+R_1)^{t-1} && \text{if } t \leq n_1 - 1 \\ &= x_1(1+R_1)^{n_1-1}(1+R_2)^{t-n_1} && \text{if } t > n_1 \end{aligned} \qquad (8.1)$$

If the aggregate growth rates in the two sub-periods are equal, i.e., $R_1=R_2=R$ then (8.1) reduces to

$$x_t = x_1(1+R)^{t-1} \tag{8.2}$$

where R is the aggregate growth rate for the entire period.

The aggregate welfare implied by sub-period growth rates R_1 and R_2 can be obtained by substituting (8.1) into (5.3) as

$$W = W[x_1,\ x_1(1+R_1)\ \ldots\ x_1(1+R_1)^{n_1-1}, x_1(1+R_1)^{n_1-1}(1+R_2),\ \ldots,\ x_1(1+R_1)^{n_1-1}(1+R_2)^{n-n_1}] \tag{8.3}$$

which must be equal to the aggregate welfare obtained by the growth rate R for the entire period (given in equation (5.7)). If the welfare function W is assumed to be homothetic, then we must have

$$W[1,(1+R)\ \ldots\ (1+R)^{n-1}] = W[1,(1+R_1)\ \ldots\ (1+R_1)^{n_1-1},\ (1+R_1)^{n_1-1}(1+R_2)\ \ldots\ (1+R_1)^{n_1-1}(1+R_2)^{n_1-n}]$$

which gives R to be a function of R_1 and R_2. It can be easily demonstrated that R is an increasing function of R_1 and R_2. R is the aggregate growth rate for the entire period (1 to n) which gives the same level of welfare as the sub-period growth rates R_1 and R_2 for the periods (1 to n_1) and (n_1 to n), respectively.

Let us assume that the welfare function is of the Form (7.1), then using some algebraic manipulations we obtain

$$\log(1+R) = \frac{(2n-n_1)(n_1-1)}{n(n-1)} \log(1+R_1) + \frac{(n-n_1)(n-n_1+1)}{n(n-1)} \log(1+R_2) \tag{8.4}$$

which gives $\log(1+R)$ to be a weighted average of $\log(1+R_1)$ and $\log(1+R_2)$. This implies that R lies between R_1 and R_2.[6] Also, it can be seen that R is an increasing function of R_1 and R_2.

R can be computed from (7.2). Similarly, R_1 can be obtained by substituting $n=n_1$ in (7.2). And, therefore, given R_1 and R we compute R_2 from (8.4).

Having computed R_1, R_2, and R, the next question that arises is whether the growth rates in the two sub-periods are significantly different? If this is so, then on can infer that there is a structural change in the economy. This issue is considered in Section 10.

---

[6]The least-squares growth rate for the whole period may lie outside the range of sub-period growth rates, or in other words, $R_1 \leq R \leq R_2$ may not hold. It means that the total growth rate may be negative (or positive) when sub-period growth rates are both positive (or negative). Our empirical investigations suggested that in 18 out of 77 countries, the total growth rate computed by LSGR procedure was outside the range of the sub-period growth rates. For instance, Bangladesh achieved an average growth rate of 1.30 percent over the period 1970-87 while the growth rate for the sub-periods 1970-79 and 1980-87 were both less than unity. Similarly, Chad achieved a negative growth rate of 2.31 during the period 1970-87 while the growth rates for the sub-periods were negative 1.88 and positive 1.48, respectively. Thus, absurdities arising from the LSGR procedure can occur frequently.

9. RELATIONSHIP BETWEEN WELFARE CHANGE AND SUB-PERIOD GROWTH RATES

Let x_1^* and x_2^* be the welfare levels (equivalent levels of incomes) in the two sub-periods (1 to n_1) and (n_1+1, n), respectively. For the symmetric welfare function (7.1), x_1^* and x_2^* are given by

$$\log x_1^* = \frac{1}{n_1} \sum_{t=1}^{n_1} \log x_t \tag{9.1}$$

and

$$\log x_2^* = \frac{1}{(n-n_1)} \sum_{t=n_1+1}^{n} \log x_t \tag{9.2}$$

respectively. Substituting (8.1) into (9.1) and (10.1) yields

$$\log x_1^* = \log x_1 + \frac{(n_1-1)}{2} \log(1+R_1)$$

$$\log x_2^* = \log x_1 + (n_1-1)\log(1+R_1) + \frac{(n-n_1+1)}{2}\log(1+R_2)$$

which gives the percentage change in welfare from period 1 to period 2 as

$$\eta = \frac{x_2^*-x_1^*}{x_1^*} = (1+R_1)^{\frac{n_1-1}{2}}(1+R_2)^{\frac{n-n_1+1}{2}} - 1 \tag{9.3}$$

where η has been defined earlier in (5.4). Thus, the percentage change in welfare is a unique function of the sub-period growth rates. It can be seen that welfare will always increase (decrease) even if $R_1=R_2=R$ provided R is positive (negative), or in other words, equality of growth rates does not imply equal welfare levels in the sub-periods. Further, note that η can be positive even if $R_2>0$ and $R_1<0$. In Table 3, for Brazil, $R_1=7.17$ percent and $R_2=-2.19$ percent which gives an increase of 23.08 percent in welfare. Even if the growth rate is negative in the second period, the welfare may still be higher. The welfare level in Brazil would have been lower in the second period only if the second period growth rate was less than -6.69 percent.

10. TESTING FOR SIGNIFICANCE OF GROWTH RATES

To test for the significance of growth rates, we need to specify a stochastic model. We begin with an identity

$$x_t = x_1(1+r_2)(1+r_3)\dots(1+r_t) \tag{10.1}$$

where r_t is the growth rate for the t^{th} period. Let R be the constant growth rate for the periods (1 to n). Substituting R for r_t into (10.1) will make the relationship an approximate one and, therefore, we introduce an error term. Thus, (10.1) becomes

$$logx_t = logx_1 + (t-1)\log(1+R) + u_t \tag{10.2}$$

where t varies from 2 to n. We assume that the error term u_t is normally distributed with a zero mean and constance variance σ^2.

One possible method of estimating R is to apply an ordinary least-squares procedure to (10.2). The growth rate obtained by this method was called RLSGR. An alternative estimator of R can be obtained by equating the sample mean of the estimated error term to zero, i.e., $\frac{1}{n}\sum_{t=1}^{n}\hat{u}_t = 0$. This procedure resulted in an estimator $\hat{R}$ of R as[7]

$$\log(1+\hat{R}) = \frac{2}{(n-1)}\,(\log x* - \log x_1) \tag{10.3}$$

where $\log x* = \frac{1}{n}\sum_{t=1}^{n}\log x_t$ is the average welfare level of the entire period.

It can be shown that $\hat{R}$ is the same as the aggregate growth rate in (7.2) which we derived using a symmetric welfare function given in (7.1). As demonstrated earlier, the RLSGR estimator implies a non-symmetric welfare function.

The above discussion demonstrates that two alternative estimators imply quite different welfare functions. Usually in econometrics, an estimator is selected on the basis of its statistical properties with no consideration given to its welfare implications. A pitfall of this procedure is that a

---

[7]This is a well-known ratio estimator widely used in econometrics in connection with errors in variable models. See A. Wald (1940).

statistically desirable estimator may be found to have undesirable welfare properties. In the present paper we have focused mainly on the welfare superior estimator of the aggregate growth rate.

Substituting (10.2) into (10.3), it is not difficult to prove that $\log(1+\hat{R})$ follows a normal distribution with mean $\log(1+R)$ and variance $\frac{4\sigma^2}{n(n-1)^2}$. To compute the standard error of $\log(1+\hat{R})$ we need to know the value of σ^2. We derived an unbiased estimator $\hat{\sigma}^2$ of σ^2 as

$$\hat{\sigma}^2 = \frac{\sum_{t=1}^{n} \hat{u}_t^2}{\left(n - \frac{2}{3} + \frac{2}{3}\,\frac{1}{(n-1)}\right)}$$

where $\hat{u}_t$ is the estimated error term given by

$$\hat{u}_t = \log x_t - \log x_1 - (t-1)\log(1+\hat{R})$$

and, therefore,

$$t = \frac{\log(1+\hat{R}) - \log(1+R)}{\frac{2\hat{\sigma}}{(n-1)\sqrt{n}}} \qquad (10.4)$$

will be distributed as the well-known "student" t distribution with $\left(n - \frac{2}{3} + \frac{2}{3}\,\frac{1}{(n-1)}\right)$ degrees of freedom. This equation can be used to test the hypothesis whether the aggregate growth rate in the period from 1 to n is statistically significantly different from a given value R.

To test for the significance of sub-period growth rates, we need to assume that the true growth rates in the sub-periods are different. Our stochastic model then becomes

$$\log x_t = \log x_1 + (t-1)\log(1+R_1) + u_{1t}, \text{ for } t \le n_1 \tag{10.5}$$

and

$$\log x_t = \log x_1 + (n_1-1)\log(1+R_1) + (t-n_1)\log(1+R_2) + u_{2t}, \text{ for } n_1 \le t \le n. \tag{10.6}$$

We assume that u_{1t} and u_{2t} are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively. If we expect a structural change in the growth performance, it is appropriate that we assume σ_1^2 to be different from σ_2^2.

We can estimate sub-period growth rates R_1 and R_2 from (10.5) and (10.6) by equating the means of estimated error terms to zero. Thus, $\hat{R}_1$ and $\hat{R}_2$ are given by the following two normal equations.

$$\log x_1^* = \log x_1 + \frac{(n_1-1)}{2}\log(1+\hat{R}_1)$$

$$logx_2^* = logx_1 + (n_1-1)\log(1+\hat{R}_1) + \frac{(n-n_1+1)}{2}\log(1+\hat{R}_2)$$

where

$$logx_1^* = \frac{1}{n_1}\sum_{t=1}^{n_1} logx_t \quad and \quad logx_2^* = \frac{1}{(n-n_1)}\sum_{t=n_1+1}^{n} logx_t$$

are the welfare levels in the first and second periods, respectively. Solving these two equations we obtain

$$\log(1+\hat{R}_1) = \frac{2}{(n_1-1)}(logx_1^* - logx_1) \tag{10.7}$$

$$\log(1+\hat{R}_2) = \frac{}{(n-n_1+1)}(logx_2^* - 2logx_1^* + logx_1) \tag{10.8}$$

Again, it can be seen that $\hat{R}_1$ and $\hat{R}_2$ are the same estimators of sub-period growth rates as obtained in Section 8 using a symmetric welfare function in (7.1). Further on substituting (10.5) and (10.6) into (10.7) and (10.8), we can demonstrate that $\log(1+\hat{R}_1)$ and $\log(1+\hat{R}_2)$ follow normal distributions with means $\log(1+R_1)$ and $\log(1+R_2)$ and variances,

$$var[\log(1+\hat{R}_1)] = \frac{4\sigma_1^2}{n_1(n_1-1)^2}$$

and

$$var[\log(1+\hat{R}_2)] = \frac{4}{(n-n_1+1)^2}\left[\frac{\sigma_2^2}{(n-n_1)} + \frac{4\sigma_1^2}{n_1}\right]$$

respectively. If we wish to test the hypothesis whether $R_2 = R_1$, we need to derive the variance of the difference; $\log(1+\hat{R}_2) - \log(1+\hat{R}_1)$, which was obtained to be equal to

$$\frac{4\sigma_2^2}{(n-n_1+1)^2(n-n_1)} + \frac{4\sigma_1^2\,(3n_1-n-3)^2}{(n-1)^2(n-n_1+1)^2 n_1}$$

Since σ_1^2 and σ_2^2 are not known, a priori, we need to estimate them. The unbiased estimators of σ_1^2 and σ_2^2 were obtained to be equal to

$$\hat{\sigma}_1^2 = \frac{\sum_{t=1}^{n_1} \hat{u}_{1t}^2}{\left[n_1 - \frac{2}{3} + \frac{2}{3}\,\frac{1}{n_1-1)}\right]}$$

and

$$\hat{\sigma}_2^2 = \frac{\left[\sum_{t=n_1+1}^{n} \hat{u}_{2t}^2 - \frac{4\hat{\sigma}_1^2(n-n_1)(n-n_1-1)}{3n_1(n-n_1+1)}\right]}{\left[(n-n_1) - \frac{2}{3} - \frac{2}{3}\,\frac{1}{(n-n_1+1)}\right]}$$

respectively, where $\hat{u}_1^t$ and $\hat{u}_2^t$ are the estimated error terms given by

$$\hat{u}_{1t} = logx_t - logx_1 - (t-1)\log(1+\hat{R}_1) \ for \ 1 \leq t \leq n_1$$

and

$$\hat{u}_{2t} = logx_t - logx_1 - (n_1-1)\log(1+\hat{R}_1) - (t-n_1)\log(1+\hat{R}_2) \ for \ n_1 + 1 \leq t \leq n.$$

Given the above information we can calculate t-statistics for testing the significance of growth rates in each sub-period and also their difference. The t-statistics follows the student t distribution, probability tables which are widely available.

11. GROWTH PERFORMANCE OF 83 DEVELOPING COUNTRIES: FURTHER ANALYSIS

Table 3 presents the growth rates for 83 countries computed using the proposed method. Since 1980, the world has plunged into the deepest and most sustained recession in the 1930s. Per capita incomes have declined in many developing countries, particularly in Africa and Latin America. Therefore, it will be appropriate to analyze the growth performance of countries in the two sub-periods 1970-79 and 1980-87. The countries were ranked according to their growth performance in each period; the higher (lower) the rank, the better (worse) the country's growth performance. These ranks are also presented in the Table.

Since the growth rates in the 1980s were generally lower than those in the 1970s, it is of interest to know whether welfare levels of countries were lower or higher in the 1980s compared to those in the 1970s. The index η given in (9.3) measures the percentage change in welfare from period 1 to period 2. This index was computed for each country on the basis of the welfare function given in (7.1). A positive (negative) value of η indicates an improvement (deterioration) in the welfare enjoyed by the countries in the 1980s to that in the 1970s.

Since we have used a homothetic welfare function, the index η is in variant with respect to the units of measurement of **x**, or in other words we could calculate, the index using per capita GNP in constant local currency. The numerical values of the index are presented in the last column of Table 3 along with its rank.

Table 4 provides a summary of Table 3. The conclusions emerging from Tables 3 and 4 are summarized below.

It will be observed that among 36 African countries, per capita growth rates have deteriorated considerably in the period 1980-87 compared to the 1970-79 period. In the 1970-79 period, there were 14 countries whose growth in GNP was less than the growth in population. This number increased to 26 in 1980-87. The actual growth rate deteriorated in 27 countries. The countries which suffered the most deterioration are Gabon, The Gambia, Kenya, Lesotho, Malawi, and Nigeria. In the 1970-79 period, Gabon enjoyed a per capita growth rate of 7.6 percent but in the 1980-87 period, per capita GNP declined at an annual rate of 11.88 percent. It is interesting to note that many African countries suffered a substantial deterioration in their incomes even in the 1970s. Some examples of such countries are Ghana, Guinea-Bissau, Madagascar, Mauritania, Niger, Uganda, and Zambia. At the same time, it may be noted that among all the developing countries, Botswana, located in Africa, enjoyed the highest growth rate of 11.38 percent. Although, its growth dropped to 5.1 percent in the 1980s, but still it was found to be one of the highest in the developing world.

As discussed in Section 9, a drop in the aggregate growth rate does not necessarily imply a drop in welfare. Although, Botswana's growth rate dropped from 11.38 percent in 1970-79 to 5.1 in 1980-87, its aggregate welfare was 103 percent higher in the 1980s compared to that in the 1970s. Other African countries which have substantially improved their welfare in the 1980s are Cameroon (65.56 percent), Congo (41.72 percent), Burkina Faso (23.11 percent), Lesotho (35.56 percent), Mauritius (23.32 percent), and Rwanda (21.95 percent). In contrast to an impressive performance of these few countries, there were large number of African countries which showed a substantial decline in their welfare in the 1980s. In 21 countries, the welfare level in the 1980s was lower than that in the 1970s. The countries which suffered the most deterioration are Gabon, Ghana, Guinea-Bissau, Liberia, Madagascar, Niger, Sudan, Uganda, Zaire, and Zambia.

Among 37 low income countries, 23 suffered a deterioration in their growth rates in the 1980s but a similar figure for the middle-income countries is 38. In 1970-79, only 2 middle-income countries had a negative growth rate; one being Chile and another Jamaica. This number increased to 25 in the 1980-87 period. From these observations we may conclude that the middle-income countries have suffered a greater decline in their standard of living than the low-income countries during the recessionary period of the 1980s. However, this conclusion does not seem to be supported if we look at the percentage change in the welfare levels (the last column in Tables 3 and 4). Nineteen of the 37 low income countries suffered a declined in welfare in the 1980s compared to that in the 1970s. But for the 46 middle-income countries, this figure was found to be only 11. Thus, growth rates alone do not tell the complete story about the changes in the standards of living.

Among 13 Middle East and Eastern Europe countries 9 suffered a deterioration in their growth rates in the 1980s. Still, for all these countries (with no exception), the welfare level in the 1980s was substantially higher than that in the 1970s. Algeria increased its welfare level by as much as 127 percent.

It is quite evident that the most Asian countries have considerably improved their growth performance during the recessionary period of the 1980s. The two most populated countries in the world, India and China, have significantly increased their growth rates. All the countries in the group improved their welfare during the 1980s. The welfare level of Korea and China was about 64 percent higher in the 1980s compared to that in the 1970s.

In the 1970s, a large number of countries in the Central and South American region enjoyed high growth rates. This performance deteriorated considerably in the 1980s. More than 80 percent of these countries registered negative growth rates in per capita GNP. The growth rate deteriorated in 21 out of 22 countries (Chile was the only country where the deterioration in growth rate did not occur). These countries were most severely affected by the debt crisis induced by high interest rates and declining demand for their exports. Many countries followed restrictive domestic fiscal and monetary policies in order to cope with external payment constraints and inflation. Although the growth rate figures portray a depressing picture of falling of standards of living, the actual drop in welfare has occurred only in nine countries. The countries which have suffered the most loss of welfare are

Bolivia, Jamaica, Nicaragua, El Salvador, and Venezuela. Many countries in Latin America have substantially increased their welfare in 1980s. Examples of such countries are Brazil (23.68 percent), Columbia (20.36 percent), Mexico (18.18 percent), Panama (20.11 percent), Paraguay (39.63 percent), and Trinidad and Tobago (18.18 percent).

An interesting question that arises is whether there has been a significant change in overall relative growth performance of countries between the 1970-79 and 1980-87 periods. This requires testing of the hypothesis that the ranking of countries according to their growth performance in 1970-79 has not altered significantly from that in 1980-87. If the relative performance of all countries was exactly the same in the two periods, the rank correlation of growth rates r in the two periods would be equal to unity. So, we need to test whether the computed value of r is significantly lower than unity. Obviously, the test statistic would be

$$t = \frac{(1 - r)}{s\ e\ (r)}$$

where s e (r) stands for error of r (see Kakwani 1988); which, under the null hypothesis of no significant difference is distributed approximately normally, with zero mean and unit standard deviation.

The value of r was computed to be 0.52 with standard error of 0.11 which gave a value of t to be equal to 4.21, which is considerably larger than 1.96. Therefore, the null hypothesis is rejected. This leads to the conclusion that the overall relative growth performance of countries has changed significantly between the 1970-79 and 1980-87 periods.

Table 3: GROWTH RATES OF GNP PER CAPITA AND PERCENTAGE IN WELFARE ESTIMATED BY THE PROPOSED METHOD

COUNTRIES	Growth Rates						% Change in Welfare	
	'1970-79		"1980-87		"1970-87		"70-79 to 80-87	
	Value	Rank	Value	Rank	Value	Rank	'Value	Rank
	AFRICA							
Burundi	0.13	18	2.19	62	0.41	28	9.45	43
Benin	-0.63	11	2.25	63	0.04	21	7.37	40
Botswana	11.38	83	5.1	75	9.87	83	103.37	82
Centr Afr Rep	0.6	24	-3.21	23	-0.31	16	-11.22	17
Cote D'Ivoire	1.86	41	-2.73	26	0.76	34	-4.08	24
Cameroon	1.51	37	10.2	82	3.49	64	65.56	80
Congo	2.89	54	5.03	73	3.39	63	41.72	71
Ethiopia	0.26	20	-0.8	42	0.01	20	-2.68	26
Gabon	7.6	79	-11.88	1	2.66	55	-21.21	9
Ghana	-1.67	5	-3.78	17	-2.17	4	-22.02	7
Guinea	1.74	39	-1.49	34	0.97	35	1.1	34
Gambia, The	4.65	65	-4.07	13	2.53	53	1.9	35
Guinea-Bissau	-1.76	4	-1.67	32	-1.74	9	-14.39	14
Burkina Faso	1.25	33	3.44	67	1.76	44	23.11	60
Kenya	6.03	73	-3.92	15	3.6	65	8.59	41
Liberia	-0.5	13	-6.52	4	-1.95	7	-27.68	3
Lesotho	8.99	81	-1.84	31	6.34	79	35.38	68
Madagascar	-1.39	8	-4.75	12	-2.19	3	-24.47	5
Mali	1.62	38	1.28	56	1.54	41	13.89	48
Mauritania	-1.5	7	0.21	50	-1.1	11	-5.64	23
Mauritius	5.84	72	-1	40	4.19	71	23.31	61
Malawi	4.85	68	-4.98	9	2.45	51	-2.05	28
Niger	-3.66	1	1.1	55	-2.56	2	-11.26	16
Nigeria	2.88	52	-6.11	6	0.69	32	-14.54	13
Rwanda	-0.56	12	5.08	74	0.74	33	21.95	58
Sudan	1.43	35	-3.78	16	0.18	25	-10.48	19
Senegal	-0.42	14	-1.18	36	-0.6	14	-6.98	22
Sierra Leone	-0.19	17	-0.27	47	-0.21	18	-2.34	27
Somalia	0.93	26	-0.93	41	0.49	29	-0.09	31
Togo	1.07	30	-2.95	24	0.11	22	-8.29	20
Tanzania	0.74	25	-3.28	21	-0.22	17	-10.99	18
Uganda	-2.67	3	-4.86	10	-3.19	1	-29.25	2
South Africa	0.97	27	-0.72	43	0.57	31	1.05	33
Zaire	-1.09	9	-5.27	8	-2.09	6	-25.43	4
Zambia	-1.67	6	-3.69	18	-2.15	5	-21.85	8
Zimbabwe	3.44	58	-2.71	27	1.96	45	2.53	37

	MIDDLE EAST AND EUROPE							
Algeria	5.09	69	14.17	83	7.16	81	127.17	83
Egypt	2.52	47	5.22	76	3.15	60	40.96	70
Greece	4.76	66	-1.14	37	3.34	62	17.07	51
Hungary	8.05	80	0	48	6.1	78	41.81	72
Jordan	2.75	49	6.76	80	3.68	67	51.33	77
Morocco	2.55	48	0.27	51	2.01	46	13.34	47
Oman	1.32	34	6.74	79	2.57	54	42.35	74
Portugal	4.37	63	-0.35	46	3.24	61	19.4	54
Syrian Arab Rep	7.05	75	-2.03	30	4.84	76	23.87	62
Tunisia	6.04	74	-0.35	45	4.5	73	28.07	64
Turkey	4.32	62	-1.05	39	3.03	59	15.32	49
Yemen Arab Rep	9.9	82	2.13	61	8.02	82	68.24	81
Yugoslavia	5.36	71	0.11	49	4.1	68	27.09	63
	ASIA							
Bangladesh	-2.77	2	5.25	77	-0.94	12	11	44
Burma	1.21	32	4.74	72	2.03	47	30.15	66
China	2.76	50	8.71	81	4.13	69	64.55	78
Indonesia	4.83	67	3.27	66	4.46	72	42.93	76
India	0.56	23	3.66	68	1.28	38	20.44	57
Korea	7.27	78	4.16	69	6.53	80	64.66	79
Sri Lanka	2.08	43	4.65	70	2.68	56	34.55	67
Malaysia	5.16	70	2.97	64	4.64	75	42.9	75
Nepal	-0.27	15	1.53	59	0.15	24	5.74	39
Pakistan	0.02	19	5.84	78	1.36	40	29.13	65
Philippines	3.59	59	-1.13	38	2.46	52	11.32	45
Thailand	3.32	56	4.69	71	3.64	66	42.24	73

	LATIN AMERICA							
Argentina	1.04	28	-3.92	14	-0.15	19	-12.49	15
Bolivia	2.86	51	-6.8	3	0.5	30	-17.3	11
Brazil	7.17	77	-2.19	28	4.89	77	23.08	59
Chile	-0.94	10	1.38	58	-0.4	15	2.03	36
Colombia	3.86	60	0.34	52	3.02	58	20.36	56
Costa Rica	3.42	57	-3.45	19	1.76	43	-0.56	29
Dominican Rep	4.16	61	-1.38	35	2.83	57	12.97	46
Ecuador	7.05	76	-3.34	20	4.51	74	16.62	50
Guatemala	3.31	55	-3.24	22	1.73	42	-0.2	30
Honduras	1.86	40	-1.66	33	1.02	36	0.68	32
Haiti	1.88	42	-0.65	44	1.28	39	5.52	38
Jamaica	-0.25	16	-5.66	7	-1.55	10	-23.98	6
Mexico	2.89	53	0.87	54	2.41	50	18.18	53
Nicaragua	0.37	21	-8.77	2	-1.86	8	-32.75	1
Panama	2.25	44	1.87	60	2.16	48	20.05	55
Peru	1.14	31	-2.05	29	0.38	27	-4.02	25
Papua New Guinea	1.06	29	-2.83	25	0.13	23	-8.04	21
Paraguay	4.56	64	3	65	4.19	70	39.63	69
El Salvador	2.51	46	-6.24	5	0.38	26	-16.51	12
Trin & Tobago	2.44	45	1.3	57	2.17	49	18.18	52
Uruguay	1.46	36	0.49	53	1.23	37	9	42
Venezuela	0.41	22	-4.76	11	-0.83	13	-18.25	10

Table 4: SUMMARY OF TABLE 3

Country Groups	Number of coun-tries	Countries with negative growth rates			Number of countries whose per capita GNP growth rate deteriorated between 1970-79 and 1980-87	Number of countries whose welfare level deteriorated between 1970-79 and 1980-87
		1970-79	1980-87	1970-87		
Low-income	37	16	22	14	23	19
Middle-income	46	2	25	5	38	11
Africa	36	14	26	13	27	21
Middle East and Europe	13	0	5	0	9	0
Asia	12	2	1	1	4	0
Latin America	22	2	15	5	21	9
Highly indebted	17	2	13	4	17	8
Others	66	16	34	15	45	22
Oil exports	13	0	5	1	9	3
Primary Producer	41	14	32	15	32	23
Manufacturing Exports	29	4	10	3	21	4
All developing countries	83	18	47	19	61	30

Having established that the overall relative growth performance of countries has changed significantly between the 1970-79 and 1980-87 periods, the next step is to see for which country groups the relative growth performance has improved (or deteriorated) in the two periods. To accomplish this task, we computed the average rank of country groups when the countries are ranked according to their growth rates. The results are presented in Table 5. The table also presents the average rank values when the countries are ranked according to the percent change in welfare between the 1970-79 and 1980-87 periods. The average rank of all 83 developing countries is always equal to 42.0. The rank average of each country group must be compared with a value of 42.0. If the average is greater (smaller) than 42.0, the relative growth performance of that country group is better (or worse) than the overall average.

It can be seen from the table that in 1970-79, the relative growth performance of the low-income countries was extremely poor with an average rank value of 27.9. However, in the 1980-87 period, their average rank value increased to 40.56 indicating a substantial increase in their relative growth performance. Accordingly, the relative growtn performance of the middle-income countries has declined substantially. The average rank of value of the low-income countries according to the welfare change is only 34.0 as against a value of 48.4 for the middle-income countries. This demonstrates that despite a deterioration in the growth performance, the middle-income countries have still managed to perform better in terms of improving their welfare level than low-income countries.

Table 5: RELATIVE GROWTH PERFORMANCE OF COUNTRIES
Classified by Income, Regions and Other Characteristics

Country Groupings	1970-79	1980-87	1970-87	Performance in welfare 1970-79 to 1980-87
Low-income	27.9	40.56	29.4	34.0
Middle-income	53.3	43.15	52.2	48.4
Africa	32.9	34.80	31.5	31.2
Middle East & Europe	63.0	155.69	66.7	65.2
Asia	42.9	67.75	52.6	63.7
Latin America	43.9	31.63	38.8	34.1
Highly indebted	46.4	28.2	40.4	34.5
Others	40.9	45.6	42.2	43.9
Oil exporters	54.6	49.6	57.2	54.1
Primary producers	31.9	32.1	29.8	30.2
Manufacturing exporters	50.6	52.6	52.4	53.2
All developing countries	42.0	42.0	42.0	42.0

12. AGGREGATION OVER COUNTRIES

So far, this paper has been focused on the aggregation of yearly growth rates of a country over a period of time. If we wish to compare the performance of countries grouped according to some socio-economic and demographic criteria, it will be necessary to aggregate growth rates over a set of countries. For instance, if we are interested in judging the growth performance of Africa, then we need to aggregate the growth rates of each country in Africa. The World Development Report (WDR), every year, presents such information for a wide range of country classifications. Difficulties associated with the procedure followed by the WDR are discussed in this section.

Let X_{it} and P_{it} be the real output (GDP or GNP in local currency but adjusted for inflation) and the population of the i^{th} country in the t^{th} year, respectively. And suppose we wish to aggregate the growth rates of m countries so that i varies from 1 to m. Since the output is measured in (constant) national currency units of each country it, as such, cannot be aggregated over countries. So one needs to convert the series in constant national currency to one in US dollars by dividing it by some fixed exchange rate. Let e_i be the exchange rate, then the total output of m countries in year t will be

$$X_t = \sum_{i=1}^{m} \frac{X_{it}}{e_i} \tag{12.1}$$

and P_t, the total population of m countries in the t^{th} year, will be

$$P_t = \sum_{i=1}^{m} P_{it} \tag{12.2}$$

where t varies from 1 to n. Then the per capita GNP of m countries will be

$$Z_t = \frac{X_t}{P_t} \tag{12.3}$$

which immediately gives their total growth rate as

$$\bar{R}_t = \frac{Z_t - Z_{t-1}}{Z_{t-1}} \tag{12.4}$$

where t varies from 2 to n. These yearly growth rates are further aggregated using the weights w_t given in (2.6). This procedure will be referred to as the WDR method.

The above procedure appears to be intuitively reasonable but it suffers from several drawbacks. First, $\bar{R}_t$ depends on the exchange rate used. It is obvious that one has to use a fixed exchange rate for all years, i.e., e_i is fixed for all t. Since exchange rates for countries vary widely over time, it raises the most difficult problem of selecting the base year. This problem is recognized by the World Bank as can be seen from the following quote:

"The national accounts commonly reported for Sub-Saharan Africa are based on 1980 prices and exchange rates, which reflect both the volume of output and economic policies in that year. On this basis, Nigeria dominates the region with a share of almost 50 percent. If 1987 exchange rates had been used following Nigeria's 86 percent current devaluation - the weights given to Nigeria would be less than 20 percent. Rather than show stagnation in Sub-Saharan Africa from 1980-87, these alternative aggregate statistics would show cumulative GDP growth of about 13 percent", "Africa's Adjustment and Growth in the 1980s" World Bank and UNDP, 1989.

Even if we assume that the problem of selecting an appropriate base year has been resolved, the use of official exchange rates is not appropriate. It is now widely recognized that the official exchange rates do not measure the relative domestic purchasing powers of currencies. The United Nations International Comparison Program (ICP) has developed measures of real GDP on an internationally comparable scale by using purchasing power parities (PPPs) instead of exchange rates, as conversion factors. These measures also suffer from a variety of methodological issues.[8] Moreover, their coverage of countries is still limited.

To measure the sensitivity of total growth rate with respect to exchange rate, we derive the elasticity of $\overline{R}_t$ with respect to e_i from (11.3):

$$E_i = \frac{e_i}{R_t}\frac{\partial \overline{R}_t}{\partial e_i} = \frac{\alpha_{it}(\overline{R}_t - r_{it})}{\overline{R}_t} \qquad (12.5)$$

[8]For an illuminating discussion of these issues see "Per Capita Income: Estimating Internationally Comparable Numbers," a World Bank document prepared by the International Economics Department, January, 1989.

where

$$\alpha_{it} = \frac{X_{i,t}}{e_i X_t} = \text{the } i^{th} \text{ country's share of total GNP, and}$$

r_{it} = growth rate of real per capita GNP of the i^{th} country in year t.

Equation (12.5) shows that η_i can be both negative and positive depending on the country's growth rate. Therefore, one cannot say, a priori, whether the aggregate growth rate will be over or under estimated because of an error in the exchange rate. Suppose a country devalues its currency by 1 percent, the aggregate growth rate will increase (decrease) if the country's growth rate is lower (higher) than the aggregate growth rate.

Secondly, E_i depends on α_{it}, the i^{th} country's share of the total GNP. Thus, the exchange rates of larger economies (the richer and larger countries) will have relatively greater impact on the aggregate growth rate.

The numerical estimates of E_i showed that the elasticity can vary widely over countries and years. The magnitude was found to vary between -2.5 to 1.8 which shows that the aggregate growth rates are highly sensitive to the changes in exchange rates. To measure this effect, we computed the aggregate growth rates for developing countries using the exchange rates prevailing in 1975, 1980, and 1985. The results are displayed in the Graph 1.

The graph shows that the aggregate growth rates for the 83 developing countries are quite close when they are measured on the basis of 1975 and 1980 exchange rates. However, the deviations become wider when we used the 1985 exchange rates. In 1979, the differences in the growth rates was as large as

1 percent. These observations suggest that any conclusions drawn from such estimates will be highly unreliable because we do not know the appropriate exchange rates or the conversion factors which would reflect the differences in purchasing power of different countries.

Equations (12.1), (12.2), and (12.3), after some algebraic manipulations give

$$(1 + \bar{R}_t) = \sum_{i=1}^{m} v_{i,t} \, (1 + r_{it}) \qquad (12.6)$$

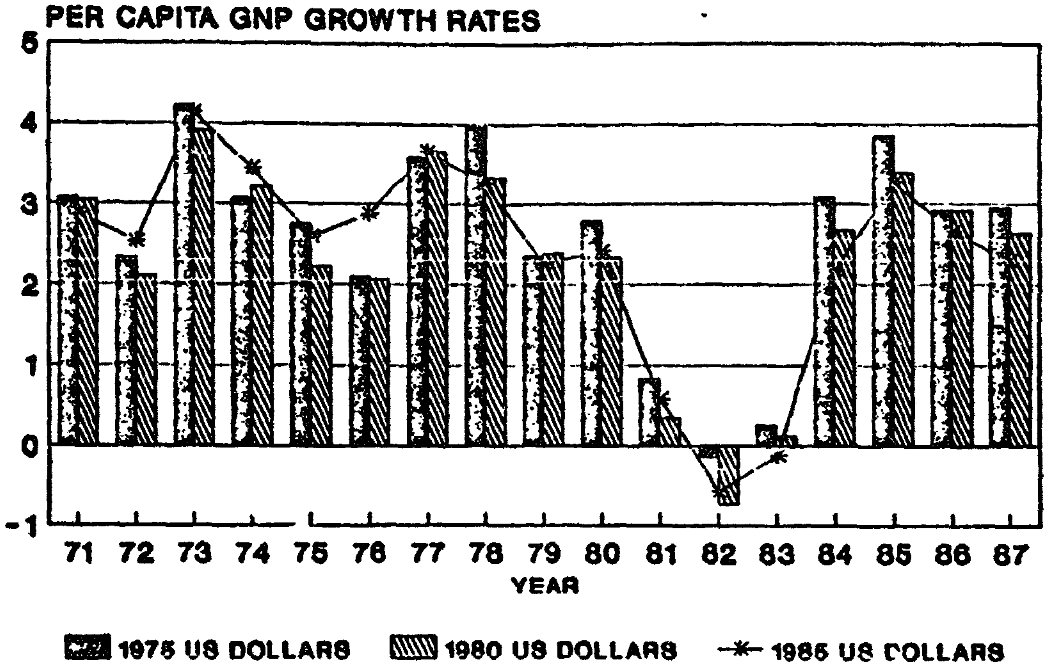
where

$$v_{it} = \frac{\alpha_{i,t-1} \, P_{it} \, P_{t-1}}{P_{i,t-1} \, P_t}$$

Assuming that the population share of a country does not change in the short-run, v_{it} becomes equal to $\alpha_{i,t-1}$, $\alpha_{i,t-1}$ being the i^{th} country's share of the total GNP in year t. This demonstrates that the growth rates of countries with higher shares of the total GNP will receive greater weights in the calculations of the aggregate growth rate. Thus, the growth rates of the richer countries (not necessarily more populated) will have a larger impact on the aggregate growth rate. Why should the growth rates of richer countries have a domineering effect on the aggregate growth rate is not clear at least from the welfare point of view? In the next section we propose an alternative procedure which does not have the drawbacks discussed above.

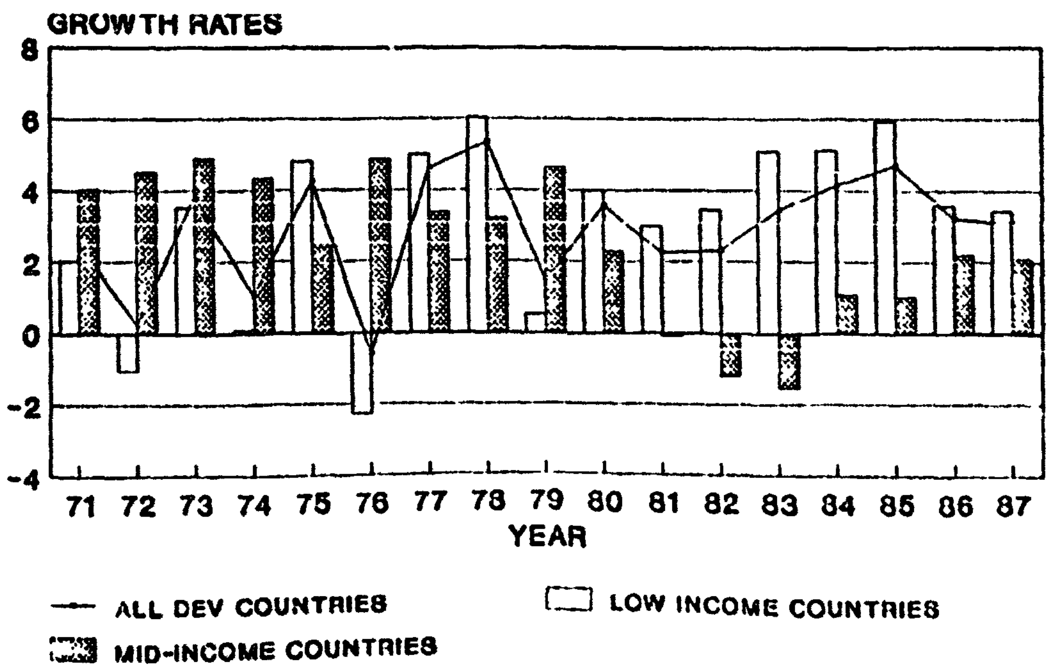
Graph 1

GROWTH RATES: ALL DEVELOPING COUNTRIES
WDR Method



Graph 2

GROWTH RATES OF PER CAPITA GNP
Using Proposed Method



13. AN ALTERNATIVE PROCEDURE

In this section we derive an aggregate growth procedure using a welfare function defined over a set of countries.

Let

$$y_{it} = \frac{X_{it}}{P_{it} e_i} \tag{13.1}$$

be the per capita GNP of the i^{th} country in year t expressed in the US dollars; e_i being the exchange rate or a conversion factor which converts GNP of a country into US dollars. This conversion to the US dollar is necessary in order to compare the utility of the i^{th} country with that of the j^{th} country. Further suppose $u(y_{it})$ is the utility enjoyed by an individual living in the i^{th} country in year t. Note that $u(y_{it})$ is comparable over time as well as across countries, provided e_i is the appropriate conversion factor.

Suppose we wish to aggregate the welfare of m countries. Then the average welfare of individuals belonging to this group of countries is given by

$$\bar{W} = \sum_{i=1}^{m} a_{it} \, u(y_{it}) \tag{13.2}$$

where $a_{it} = \frac{p_{it}}{\left(\sum_{i=1}^{m} p_{it}\right)}$ is the population share of the ith country in year t. Let

$\bar{y}_t$ be t e equivalent per capita GNP which, if distributed equally among the countries in the group, will result in the actual average welfare level.

Since $\sum_{i=1}^{m} a_{it} = 1.0$, $\bar{y}_t$ must satisfy

$$u(\bar{y}_t) = \sum_{i=1}^{m} a_{it}\, u(y_{it}) \tag{13.3}$$

so that $\bar{y}_t$ can be used as an indicator of welfare of m countries.

We may now define the aggregate growth of m countries in year t as

$$\bar{r}_t = \frac{(\bar{y}_t - \bar{y}_{t-1})}{\bar{y}_{t-1}} \tag{13.4}$$

which can be seen to be invariant with respect to a positive linear transformation of the utility function.

To compute $\bar{r}_t$ we need to specify the utility function. A simplest function which satisfies all the desirable properties of a utility function is the logarithmic function. Using the logarithmic function in conjunction with (13.3) and (13.4) we obtain

$$\log(1+\bar{r}_t) = \sum_{i=1}^{m} a_{i,t-1}[\log(y_{it}) - \log(y_{i,t-1})] \tag{13.5}$$

where it assumed that $a_{i,t-1}$ will be approximately equal to $a_{i,t}$ or in other words the population share of the i^{th} country remains approximately the same in year (t-1) and t.

Recall that r_{it} is the growth of the i^{th} country in year t which is computed from the per capita GNP series in the constant local currency. It is given by

$$\left(\frac{X_{it}}{P_{it}}\right) = (1+r_{it})\left(\frac{X_{it-1}}{P_{i,t-1}}\right)$$

which on dividing by e_i both sides gives

$$y_{it} = (1 + r_{it})\, y_{i,t-1} \tag{13.6}$$

which on substituting in (13.5) gives

$$\log(1+\bar{r}_t) = \sum_{i=1}^{m} a_{i,t-1} \log(1+r_{it}) \tag{13.7}$$

which provides a method of computing the aggregate growth rate of a group countries. Note that the exchange rate does not enter at all in the calculation of $\bar{r}_t$. The logarithmic utility function is the only function which makes the aggregate growth rate independent of the exchange rate.

It can be seen that the WDR method is based on the linear utility function; $u(y_{it}) = y_{it}$ which ignores completely the inequality of incomes across the countries. The population weights in (13.7) imply that the welfare performances of all individuals get equal weights irrespective of which country they belong to. The proposed procedure is simple to compute; it does not require the use of exchange rate and is based on a meaningful welfare function.

14. A COMPARISON OF GROWTH PERFORMANCE OF COUNTRIES BY ALTERNATIVE CLASSIFICATIONS

This section provides an analysis of growth performance of countries classified by income, regions and whether the countries are oil exporters, highly indebted and primary producers. The main purpose will be to compare growth rates computed by the WDR method and the alternative procedure proposed in the previous section. As discussed in the previous section, the WDR procedure of aggregating growth rates over countries depends on what exchange rates are used for each country. In this section we computed the aggregate growth rates by using constant 1985 US dollars official exchange rates for each country. The alternative method proposed in the paper does not require the use of exchange rates at all.

The annual growth rates for the 83 developing countries, computed by the WDR method were presented in Graph 1. Graph 2 presents the annual growth rates computed by the proposed procedure. This graph also presents growth rates for low and middle income countries.

The graphs show that the two methods can give widely different results. For instance in 1982 the WDR procedure showed a negative growth rate of about 0.5 percent for the 83 developing countries reported in the study whereas the proposed method suggested that the same countries had a positive growth rate of about 2 percent. The divergence in growth performances by the two methods is widest between 1980 and 1984. The proposed method indicates that the growth performance of developing countries in the 1980s is not as dismal as is shown by the WDR method.

The yearly growth rates were further aggregated over the periods 1970-79, 1980-87, and 1970-87. The results are presented in Table 6. This Table also gives the percentage change in welfare level between periods from 1970-79 to 1980-87.

This table also indicates that the overall growth performance of developing countries in the 1980s is not as dismal. The proposed method suggests that these countries have in fact had a higher growth rate in the 1980s than in the 1970s. The overall growth rate in the 1970-79 period was 2.26 percent which increased to 3.71 percent in the 1980-87 period. The WDR method gives an opposite result; it shows a drop in growth rate from 3.10 to 1.42 percent.

The impact of the economic crisis of the 1980s has not been uniform. Some regions have been more seriously affected than others. The Asian countries have substantially improved their growth rates whereas all other regions have suffered severe deterioration. Africa has suffered the most

Table 6: PER CAPITA GROWTH RATES OF VARIOUS COUNTRY CLASSIFICATIONS

Country Grouping	Number of Countries	WDR Method			Proposed Method			Percentage Change in Welfare Between 1970-79 to 1980-87
		1970-79	1980-87	1970-87	1970-79	1980-87	1970-87	
Low-income	37	2.15	3.45	2.69	1.69	4.79	2.41	82.89
Middle-income	46	3.52	0.08	1.81	4.12	0.27	3.20	50.56
Africa	36	1.09	-2.33	-0.63	1.19	-2.87	0.22	-15.15
Middle East & Eastern Europe	13	4.31	0.91	2.57	4.53	2.09	3.95	85.41
Asia	12	3.15	4.77	3.89	1.97	5.85	2.87	106.56
Latin America	22	3.08	-1.11	1.04	3.99	-1.67	2.63	23.58
Highly Indebted	17	3.04	-1.54	0.80	3.84	-2.22	2.38	15.57
Others	66	3.04	3.36	3.14	1.97	4.85	2.64	88.75
Oil Exporters	13	3.27	-0.97	1.42	3.71	1.18	3.11	58.01
Primary Producers	41	0.66	-1.91	-0.60	0.98	-1.67	0.35	-6.51
Manufacturing - Exporters	29	3.48	3.04	3.09	2.22	5.1	2.89	97.61
All Developing Countries	83	3.10	1.42	2.23	2.26	3.71	2.60	74.78

deterioration in the growth rate; suggesting a decline in per capita GNP at an annual rate of 2.87 percent in the 1980s, where the similar figure for Latin American countries as a group is 1.67 percent.

The last column in the table indicates that in the 83 developing countries as a group, the welfare level has increased by 74.78 between 1970-79 and 1980-87. The percentage increase is larger among the low-income countries compared to that among the middle-income countries.

In the previous section we observed that the middle-income countries performed relatively better than the low-income countries in terms of their welfare levels.

This relative performance of the middle-income countries was superior because each country received equal weight irrespective of its size. The growth rates computed in Table 6 give equal weight to each individual, irrespective of which country that individual belongs. Since the more populated countries of Asia performed extremely well in the 1980s, therefore, the over-all performance of the low-income countries was found to be superior.

15. CONCLUSIONS

This paper has been concerned with the measurement of aggregate growth rates. The aggregation is considered over time as well as over countries. The paper demonstrates that the mechanical procedures of computing aggregate growth rates have welfare implications. However, the value judgments implied by them were not found to be intuitively natural. A new procedure is suggested which captures all the desirable properties of a welfare function. Some of the conclusions emerging from the study are summarized below.

The empirical results show that a large number of countries fell victim to the global recession in the 1980s. Forty-seven out of 83 developing countries registered negative growth rates of per capita GNP between 1980 and 1987. Most of these countries are located in Sub-Saharan Africa and Latin America. The Asian countries have considerably improved their growth performance during the 1980s. The two most populated countries in the world, India and China have significantly increased their growth rates.

The results indicate that a drop in the aggregate growth rate does not necessarily imply a drop in welfare. Although, Botswana's growth rate dropped from 11.39 percent in 1970-79 to 7.72 in 1980-87, its aggregate welfare was 103 percent higher in the 1980s compared to that in the 1970s. In the Central and South African regions, the growth rate deteriorated in 21 out of 22 countries between 1970-79 and 1980-87, the actual drop in welfare occurred only in 9 countries. In fact, many countries in Latin America substantially increased their average welfare in the 1980s.

The overall relative growth performance of all developing countries changed significantly between the 1970-79 and 1980-87 periods. The low-income countries registered a substantial improvement in their relative growth performance. Although the relative growth performance of the middle-income countries deteriorated considerably in the 1980s, they still managed to improve their relative welfare levels better than the low-income countries.

The proposed method of aggregating growth rates over countries showed that the overall growth performance of developing countries has not been as dismal as it has been thought to be. These countries have in fact had a higher growth in the 1980s than in the 1970s. The overall growth rate in the 1970-79 period was 2.26 percent which increased to 3.71 percent in the 1980-87 period. The World Bank method gives an opposite result, it shows a drop in growth rate from 3.10 to 1.42 percent.

Finally, a concluding remark. This paper has been focused on measuring growth rates of per capita GNP. This methodology should be extended to measure a country's performance in other indicators of welfare such as life expectancy at birth or infant mortality rate. These indicators pose special problems which will be considered in our on-going work on the standards of living.

REFERENCES

Atkinson, A.B. (1970), "On the Measurement of Inequality", Journal of Economic Theory, Volume 2, pp. 244-263.

Boyce, James (1986), "Kinked Exponential Models for Growth Rates Estimation", Oxford Bulletin of Economics and Statistics, Volume 48, Number 4, pp. 385-391.

Hadar, J. and W.R. Russell (1969), "Rules for Ordering Uncertain Prospects", American Economic Review, Volume 55, pp. 25-34.

Hanock, G. and H. Levy (1969), "The Efficiency Analysis of Choices Involving Risk", Review of Economic Studies, Volume 36, pp. 334-46.

Kakwani, Nanak (1981), "Welfare Measures: An International Comparison", Journal of Development Economics, Volume 8.

Quirk, J.P. and Saposnik, R. (1962), "Admissibility and Measurable Utility Functions", Review of Economic Studies, Volume 29, pp. 140-146.

Rothschild, M. and J.E. Stiglitz (1973), "Some Further Results on the Measurement of Inequality", Journal of Economic Theory, Volume 6, pp. 188-204.

Rothschild, M. and J.E. Stiglitz (1970), "Increasing Risk: A Definition", Journal of Economic Theory, Volume 2, pp. 225-243.

Sen, Amartya (1985), "The Standard of Living", 1985 Tanner Lecturers given at Clare Hall, Cambridge.

Sen, Amartya (1974), "International Bases of Alternative Welfare Approaches: Aggregation and Income Distribution", Journal of Public Economics, Volume 4, pp. 387-403.

Sen, Amartya (1973), "On Economic Inequality", Oxford: Clarendon Press.

Wald, A. (1940), "The Fitting and Straight Lines of Both Variables are Subject to Error," Ann. Math. Statist, Vol. 11, pp. 284-300.

World Bank (1989), "Africa's Adjustment and Growth in the 1980s", The World Bank and the UNDP.

World Bank (1989), "Per Capita Income: Estimating Internationally Comparable Numbers", International Economics Department.

	Title	Author	Date	Contact for paper
WPS618	Poverty and Development: The *Human Development Report* and the *World Development Report*, 1990	Ravi Kanbur	March 1991	J. Sweeney 31021
WPS619	Foreign Direct Investment in Sub-Saharan Africa	Laurence Cockcroft Roger C. Riddell	March 1991	S. King-Watson 33730
WPS620	Have Commercial Banks Ignored History?	Sule Özler	March 1991	S. King-Watson 33730
WPS621	Sensible Debt Buybacks for Highly Indebted Countries	Enrica Detragiache	March 1991	S. King-Watson 33730
WPS622	How Factors in Creditor Countries Affect Secondary Market Prices for Developing Country Debt	Sule Özler Harry Huizinga	March 1991	S. King-Watson 33730
WPS623	World Bank-Supported Adjustment Programs: Country Performance and Effectiveness	Vittorio Corbo Patricio Rojas	March 1991	A. Oropesa 39075
WPS624	Choosing Policy Instruments for Pollution Control: A Review	Gunnar S. Eskeland Emmanuel Jimenez	March 1991	A. Bhalla 37699
WPS625	How Trade and Macroeconomic Policies Affect Economic Growth and Capital Accumulation in Developing Countries	Ramon Lopez	March 1991	K. Cabana 37946
WPS626	The Macroeconomics of the Public Sector Deficit: The Case of Colombia	William Easterly	March 1991	R. Luz 39059
WPS627	The Role of Institutions in Poverty Reduction: A Focus on the Productive Sectors	Sharon L. Holt	March 1991	WDR Office 31393
WPS628	The Indonesian Family Planning Program: An Economic Perspective	Dov Chernichovsky Henry Pardoko David De Leeuw Pudjo Rahardjo Charles Lerman	March 1991	O. Nadora 31091
WPS629	An Atheoretic Evaluation of Success in Structural Adjustment	Patrick Conway	March 1991	D. Ballantyne 37947
WPS630	The Spirit of Capitalism and Long-run Growth	Heng-fu Zou	March 1991	A. Bhalla 37699
WPS631	The Macroeconomics of the Public Sector Deficit: The Case of Morocco	Riccardo Faini	March 1991	R. Luz 34303

PRE Working Paper Series

	Title	Author	Date	Contact for paper
WPS632	The Macroeconomics of the Public Sector Deficit: The Case of Argentina	Carlos Alfredo Rodriguez	March 1991	R. Luz 34303
WPS633	The Macroeconomics of the Public Sector Deficit: The Case of Thailand	Virabongse Ramangkura Bhanupongse Nidhiprabha	March 1991	R. Luz 34303
WPS634	Trends in Developing Country Exports, 1963-88	Bela Balassa	March 1991	WDR Office 31393
WPS635	Exchange Rates and Foreign Trade in Korea	Bela Balassa	March 1991	C. Cuskelly-Young 39413
WPS636	Economic Integration in Eastern Europe	Bela Balassa	March 1991	C. Cuskelly-Young 39413
WPS637	Poverty in Poland, 1978-88	Branko Milanovic	March 1991	WDR Office 31393
WPS638	Researching the Trade-Productivity Link: New Directions	James Tybout	March 1991	D. Ballantyne 37947
WPS639	The High Cost of Protecting Uruguay's Automotive Industry	Wendy E. Takacs	March 1991	D. Ballantyne 37947
WPS640	The Impact of Policy in African Agriculture: An Empirical Investigation	William Jaeger	March 1991	A. Yideru 34663
WPS641	Intertemporal Substitution in Consumption: Evidence for Some High- and Middle-Income Countries	Karsten N. Pedersen	April 1991	M. Hileman 31284
WPS642	How a Change in Brazil's Sugar Policies Would Affect the World Sugar Market	Brent Borrell	April 1991	A. Kitson-Walters 33712
WPS643	Regional Integration among Developing Countries, Revisited	Andras Inotai	April 1991	D. Ballantyne 37947
WPS644	Trade and Payments Arrangements in Post-CMEA Eastern and Central Europe	Constantine Michalopoulos David Tarr	April 1991	M. Colinet 34698
WPS645	Poverty, Policy, and Industrialization: Lessons from the Distant Past	Ben Polak Jeffrey G. Williamson	April 1991	WDR Office 31393
WPS646	The Developmental Effectiveness of Aid to Africa	Tony Killick	April 1991	S. King-Watson 33730
WPS647	Growth Rates and Aggregate Welfare: An International Comparison	Nanak Kakwani	April 1991	B. Rosa 33751